

CP-Violating Solitons in the Early Universe*

Ola Törnkvist[‡] and Antonio Riotto[§],

*NASA/Fermilab Astrophysics Center,
Fermi National Accelerator Laboratory,
Batavia, Illinois 60510-0500, USA*

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Abstract

Solitons in extensions of the Standard Model can serve as localized sources of CP violation. Depending on their stability properties, they may serve either to create or to deplete the baryon asymmetry. The conditions for existence of a particular soliton candidate, the membrane solution of the two-Higgs model, are presented. In the generic case, investigated by Bachas and Tomaras, membranes exist and are metastable for a wide range of parameters. For the more viable supersymmetric case, it is shown that the present-day existence of CP-violating membranes is experimentally excluded, but preliminary studies suggest that they may have existed in the early universe soon after the electroweak phase transition, with important consequences for the baryon asymmetry of the universe.

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[‡]Address after 24 Sept. 1997: DAMTP, Univ. of Cambridge, Silver Street, Cambridge CB3 9EW, England. Email: olat@fnal.gov

[§]PPARC Advanced Fellow, Oxford Univ. from Sept. 1997. From 1 Dec. 1997 on leave of absence at CERN, Theory Division, CH-1211 Geneva 23, Switzerland. Email: riotto@fnal.gov

In this talk I report on work done in collaboration with Antonio Riotto[1] on CP-violating solitons and their possible consequences for the baryon asymmetry of the Universe. The talk is organized as follows: After reviewing some well-known sources of CP violation in models of particle physics, I propose that solitons in extensions of the Standard Model can serve as *localized* sources of CP violation. From then on I limit the discussion to the type of soliton considered in Ref. [1] – membranes. The conditions for existence of CP-violating membranes are presented in two important cases, corresponding to the smallest possible extensions of the Standard Model: (1) a generic model with two Higgs doublets, and (2) the supersymmetric two-Higgs-doublet model, better known as the Minimal Supersymmetric Standard Model (MSSM). Finally, I discuss the implications of the existence of CP-violating solitons in the early Universe and argue that they may, depending on their stability properties, serve either to create or to deplete the baryon asymmetry.

Sources of CP Violation

In the Standard Model (SM) of particle physics, CP non-conserving processes are the result of an irreducible complex phase in the Cabibbo-Kobayashi-Maskawa matrix, which describes the mixing of quark flavors of different generations. Because this phase enters only in higher-loop processes involving massive charged W bosons, CP violation in the SM is strongly suppressed. Except in highly speculative scenarios, it cannot account for the observed baryon asymmetry of the Universe [2].

Extensions of the SM with two Higgs doublets H_1 and H_2 provide an additional source of CP violation in the relative phase δ of their expectation values in the ground state,

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}; \quad \langle H_2 \rangle \equiv e^{i\delta} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (1)$$

A non-zero value of δ arises from loop corrections to the Higgs potential that determines the minimum. At finite temperature, the corrections can be large and lead to “spontaneous” CP violation [3] as H_1 , H_2 acquire expectation values in the electroweak phase transition. It is generally agreed that extended models of particle physics provide sources of CP-violation that are sufficiently strong for the purpose of baryogenesis.

Localized Sources of CP Violation

For the sake of simplicity take $\delta = 0$. Then the ground state (1) contains no spatially uniform source of CP violation. However, solitonic excitations exist that include a phase

δ which is non-zero only in a localized region of space. In such a case, CP violation is confined to the interior of the soliton.

As a first example, consider a texture [Copeland]¹ of the form

$$H_1 = \begin{pmatrix} 0 \\ f_1(\mathbf{x}) \end{pmatrix}; \quad H_2 = U(\mathbf{x}) \begin{pmatrix} 0 \\ f_2(\mathbf{x}) \end{pmatrix}, \quad (2)$$

where $U(\mathbf{x}) \in SU(2)$ and $\pi_3[SU(2)] = \mathbf{Z}$. A spherical solution of this form has been investigated by Bachas et al. [4], who found it to be unstable for all parameters of the model.

For the rest of the talk I will concentrate on *membranes*, which are wall-like solutions similar to domain walls. The important difference is that a membrane, much like a soap film, separates two regions that are in the identical state, whereas a domain wall separates different states. Viewed in a local rest frame, the flat membrane is a static solution depending only on the coordinate x orthogonal to the wall:

$$H_1 = \begin{pmatrix} 0 \\ f_1(x) \end{pmatrix}; \quad H_2 = e^{i\theta(x)} \begin{pmatrix} 0 \\ f_2(x) \end{pmatrix}, \quad (3)$$

where $\theta(-\infty) = 0$ and $\theta(\infty) = 2\pi$. The phase θ changes from 0 to 2π within a distance synonymous with the thickness of the membrane M_A^{-1} , where M_A is the mass of the lightest CP-odd neutral Higgs boson. Therefore, $e^{i\theta} \neq 1$ inside the membrane while $e^{i\theta} = 1$ outside.

The presence of such a kink solution can be easily understood by noticing that the Lagrangian of the model contains terms similar to those of the sine-Gordon model Lagrangian. In fact, using the form (3) and the approximation $f_1 \approx v_1$, $f_2 \approx v_2$ the terms $|D_\mu H_1|^2 + |D_\mu H_2|^2 + m_3^2 [H_1^\dagger H_2 + H_2^\dagger H_1]$ reduce to² $v_1 v_2 [\cos \beta \sin \beta (\theta')^2 + 2m_3^2 \cos \theta]$. Here $\tan \beta = v_2/v_1$ is the usual constant parameter defined by the ratio of the two expectation values. The corresponding potential $V \propto -\cos \theta$ is known to give rise to kink solutions that interpolate from one minimum at $x = -\infty$ to the adjacent minimum at $x = +\infty$. The analytic solution is the sine-Gordon kink, given by $\theta(x) = 4 \tan^{-1}[\exp(M_A x)]$, where $M_A^2 = m_3^2/(\cos \beta \sin \beta)$ determines the characteristic thickness. This solution is identical to the time evolution $\theta(t)$ of the angle of a pendulum released at rest from its highest vertical position and given a small kick so that it swings through a full circle within a time $\sim M_A^{-1}$ and comes to rest again asymptotically in its highest position as $t \rightarrow +\infty$.

¹A single name in brackets refers to that person's contribution to these proceedings.

²Care must be taken to solve for the gauge potential in terms of θ' [1].

Conditions for the Existence of Membranes

Let me first discuss the generic model with two Higgs doublets. In this case the Higgs sector contains 9 parameters, and the membrane solutions of this model were investigated by Bachas and Tomaras [5]. They restricted themselves to a hyperplane in parameter space by fixing 6 of the parameters, including $\tan\beta (= 1)$. For the remaining 3 parameters they picked M_{h^0}/M_A , M_{H^0}/M_A and M_{H^\pm}/M_A , where h^0 , H^0 , H^+ and H^- are the CP-even Higgs bosons of the model, and were able to show that there exist *classically stable* membranes when $M_{h^0}/M_A \gtrsim 2$, $M_{H^\pm}/M_A \gtrsim 2.2$ and $M_{H^0}/M_A \gtrsim g(M_{h^0}/M_A)$, g being a monotonically decreasing function. I want to emphasize that membranes thus exist in a large region of parameter space, especially since 6 of the parameters were fixed in this study.

Because these solitons are non-topological, classical stability means that they constitute a local minimum of the energy, that they are *metastable*, and that they will decay after a finite life-time via the process of tunneling.

I now proceed to the case of the supersymmetric two-Higgs model, or the MSSM. This model is, of course, much more attractive from the particle physics point of view, since supersymmetry is a major motivation for having two Higgs doublets in the first place. Supersymmetry provides ways to solve many of the puzzles of the SM such as the electroweak symmetry breaking, the problem of quadratic divergences in the Higgs mass, and stability of the weak scale under radiative corrections without fine-tuning. In addition, supersymmetric particles serve as candidates for dark matter [Sadoulet].

Together with Antonio Riotto, I have shown that there exist no membrane solutions in the MSSM for any parameters unless one includes one-loop quantum corrections to the potential [1]. These corrections are highly significant because supersymmetry leads to accidental conspiracies in the classical tree-level potential. In order to address the existence of membranes, one therefore *must* consider the one-loop corrected potential. The low-energy limit of this model can be described by three parameters, M_A , $\tan\beta$, and M_{SUSY} , while the rest of the parameters are constrained by supersymmetry. Here M_{SUSY} is the scale of supersymmetry breaking, typically in the range $M_W \ll M_{\text{SUSY}} \lesssim \mathcal{O}(\text{few})$ TeV.

The issue of existence of membrane solutions hinges crucially on whether the magnitudes f_1 and f_2 of eq. (3) are able to satisfy their boundary conditions $f_i = v_i$ at $x = \pm\infty$. This turns out to require a low value of the mass M_A , corresponding to thick membrane walls. Fig. 1 shows our results [1] for the region of parameter space $(M_A, \tan\beta)$ where solutions exist, for one “low” and one “high” value of M_{SUSY} . We cannot choose M_{SUSY} much higher without reintroducing the need for fine-tuning. Also in the figure are two bounds. The vertical bound is from LEP measurements of $e^+e^- \rightarrow h^0 A^0, h^0 Z$, requiring

$M_A \gtrsim 62.5$ GeV for $\tan \beta > 1$. The horizontal bound results from assuming that physics is described by a Grand Unified Theory (GUT) at a scale of about 10^{16} GeV. In such a case the value of the top Yukawa coupling h_t at low energies is predicted by the theory, and combining this with the measured value of the top-quark mass $M_t = (175 \pm 6)$ GeV implies that $\tan \beta \gtrsim 1.1$. This minimal value corresponds to a gauge-mediated supersymmetry-breaking mechanism.

From Fig. 1 it is apparent that no CP-violating membranes exist in the MSSM at zero temperature (for which our analysis was done), provided that one abides with the GUT hypothesis.

One may now ask oneself whether the non-existence of membranes at zero temperature is in fact an attractive feature, and whether membranes could have existed at temperatures near that of the electroweak phase transition, $T_c \sim 10^2$ GeV, and vanished at low temperatures. Our preliminary investigations show that such a scenario is possible and would result from large finite-temperature quantum corrections to M_A^2 , the parameter that most directly influences the existence of the membranes. Taking $M_{A,T=0}^2$ slightly larger than the experimental limit of $(62.5)^2 \text{ GeV}^2$, the question is whether one can suppress the high-temperature value $M_{A,T>0}^2 = M_{A,T=0}^2 + \delta M_{A,T>0}^2$ by means of a sufficiently large negative correction $\delta M_{A,T>0}^2$ so as to bring it down to values $\lesssim 20^2 \text{ GeV}^2$ required for membrane solutions.

The dominant contribution to $\delta M_{A,T>0}^2$ comes from stop loops and is given by [6] $\delta M_{A,T>0}^2 = \mu A_t h_t^2 f(\frac{M_Q}{T}, \frac{M_U}{T}) / (\sin \beta \cos \beta)$, where

$$f(x, y) \equiv -\frac{3}{\pi^2} \left[\frac{J'_+(x^2) - J'_+(y^2)}{x^2 - y^2} + \frac{\pi}{4} \left(\frac{1}{x+y} - \frac{1}{\tilde{x} + \tilde{y}} \right) \right] \quad (4)$$

and $J_+(x^2) \equiv \int_0^\infty dt t^2 \ln[1 - \exp(-\sqrt{t^2 + x^2})]$. Here μ , A_t , M_Q and M_U are parameters in the stop mass matrix, and a tilde (\sim) implies the use of 1-loop self-energy corrected masses \tilde{M}_Q and \tilde{M}_U [6]. For a concrete example, take $-\mu = A_t = M_Q = M_U = T = 145$ GeV and $\tan \beta = 1$. We then find $f(1, 1) = .0433 \dots$, $\delta M_{A,T>0}^2 \approx -3644 \text{ GeV}^2$, and $M_{A,T>0}^2 \approx 16^2 \text{ GeV}^2$. This indicates that membranes may exist in the MSSM at $T \lesssim T_c$. Riotto and I are currently investigating this issue more carefully.

Implications for the Baryon Asymmetry

In electroweak models, the chiral anomaly permits processes at high temperature that do not conserve the baryon (B) and lepton (L) numbers, but conserve $B - L$ [2]. These B -violating *sphaleron* transitions involve passing over an energy barrier of height $E_{\text{sph}} \propto (|H_1|^2 + |H_2|^2)^{1/2}$, where $E_{\text{sph}} \approx 10$ TeV at zero temperature. The B -violating transition probability is proportional to $\exp(-E_{\text{sph}}/T)$.

As we can see from Fig. 2, the magnitudes $f_i = |H_i|$, $i = 1, 2$, decrease inside a membrane, leading to a lower barrier and higher transition rate. Moreover, our results indicate that $M_A \ll M_W$ in order for membranes to exist. This means that the membrane is thick enough to accommodate the field configuration at the top of the barrier, the sphaleron, which has a characteristic size of M_W^{-1} .

From now on let us assume that membrane solutions do exist at high temperatures. The picture I want to present features membranes forming inside expanding critical bubbles in a first-order phase transition [Gleiser]¹ at temperatures just below the critical temperature. The membranes are bounded by string loops, in analogy with axionic walls [Sakharov], or (more rarely) form closed surfaces. Because the size of a critical bubble is much larger than M_A^{-1} , many membranes form inside a bubble.

The implications of these membranes are different depending on their stability properties. If they are metastable, they can move across the plasma and create non-equilibrium conditions as they strike particles in their way. Since CP is violated in the interior of the membrane, and the B -violation rate is enhanced there, all three Sakharov conditions [Riotto] may be satisfied, and a net $B(= L)$ can be generated that will survive if the phase transition is strongly enough first-order.

If, on the other hand, the membranes are unstable, they will still be created as thermal fluctuations inside a critical bubble. The nucleation rate for a size- R membrane is of the order of $T \exp[-F(R)/T]$. In order for membranes to play a role, this rate must be larger than the expansion rate of the universe, $H = T^2/M_{\text{Pl}}$, which is true if $F(R)/T \lesssim 40$. This means that small membranes are ubiquitous as fluctuations and provide the conditions for B and CP violation in the critical bubble. Because this scenario occurs in thermal equilibrium, no net $B + L$ can be generated. The membranes can, however, serve to deplete pre-existing baryon asymmetry.³

Conclusions

In summary, I have demonstrated that both the generic and the supersymmetric two-Higgs Standard Model contain CP-violating soliton solutions. If Nature is described by a generic two-Higgs model, then there exist metastable membranes that provide a new mechanism for creating the baryon asymmetry of the universe. If Nature is supersymmetric, such membranes are excluded at zero temperature, but may exist (unstable or metastable) near the electroweak phase transition temperature.

³This does not affect the asymmetry in models of GUT baryogenesis where a net $B - L$ is created at the GUT scale.

FIGURES

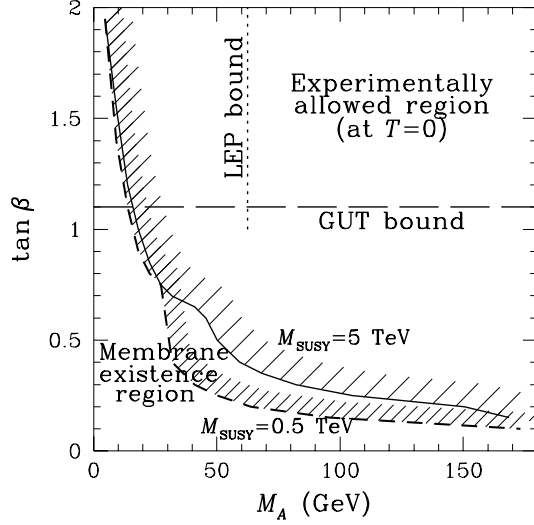


FIG. 1

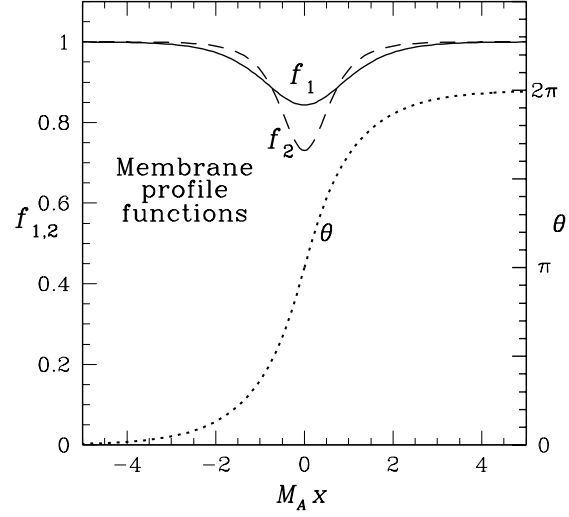


FIG. 2

I emphasize that CP-violating solitons, whatever their particular realization, can play an important role in critical bubbles immediately after a first-order phase transition, serving either to create or deplete the baryon asymmetry.

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REFERENCES

- [1] A. Riotto and O. Törnkvist, *CP-Violating Solitons in the Minimal Supersymmetric Standard Model*, hep-ph/9704371, to be published in Phys. Rev. **D**.
- [2] For a recent review, see for instance, V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk. **166**, 493 (1996).
- [3] D. Comelli, M. Pietroni, Phys. Lett. **B306** (1993) 67; D. Comelli, M. Pietroni, A. Riotto, Nucl. Phys. **B412** (1994) 441; J.R. Espinosa, J.M. Moreno, M. Quirós, Phys. Lett. **B319** (1993) 505.
- [4] C. Bachas, P. Tinyakov, T.N. Tomaras, Phys. Lett. **B385** (1996) 237.
- [5] C. Bachas and T.N. Tomaras, Phys. Rev. Lett. **76** (1996) 356.
- [6] A. Brignole, J.R. Espinosa, M. Quiros, F. Zwirner, Phys. Lett. **B324** (1994) 181.